

# **Introduction: from networks to graphs**

Nicolas Tremblay



# What is a network?

Within a given **context**, a collection of **interconnected objects**

# What is a network?

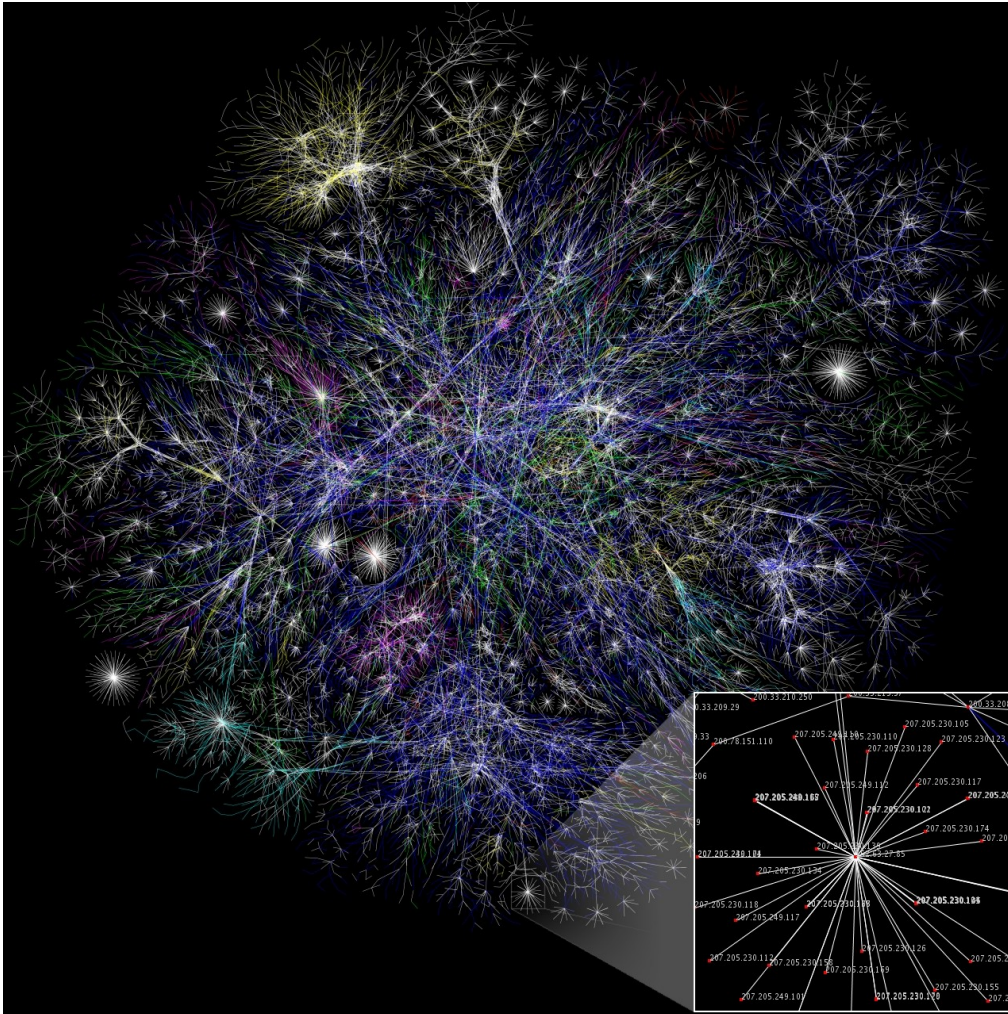
Within a given **context**, a collection of **interconnected objects**

## A **social** network



- **Objects**: Facebook profiles
- **Interconnections**: “friend” status

# What is a network?



## The **Internet** network

- **Objects:** IP addresses
- **Interconnections:** physical cable

# What is a network?

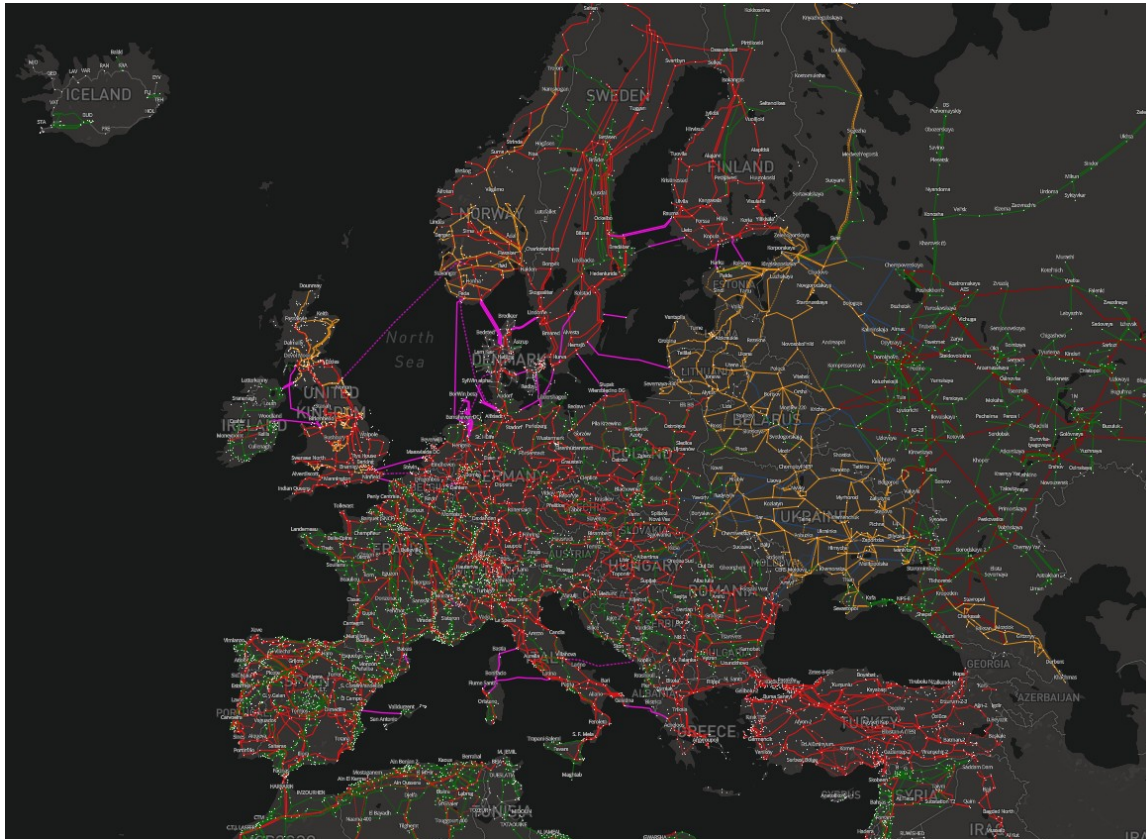
## A transportation network



- **Objects:** Grenoble TRAM stops
- **Interconnections:** TRAM rail



# What is a network?

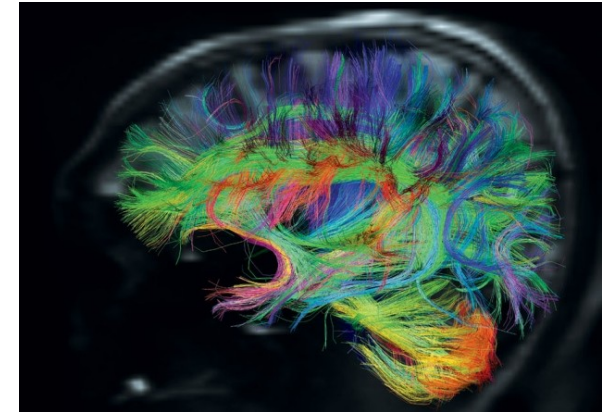


## The **electrical** network

- **Objects**: power plants / converters
- **Interconnections**: physical cable

# What is a network?

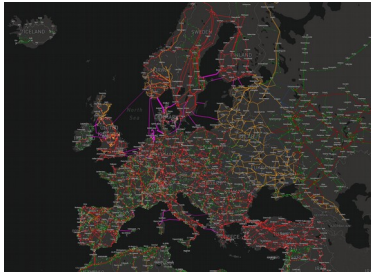
## A brain network



- **Objects**: neurons or brain regions
- **Interconnections** (example):
  - white matter physical connections
  - or functional correlations

# Why bother studying networks?

Understand the link between the ***structure*** of the network and...

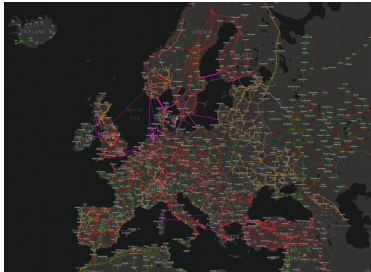


*Efficiency*



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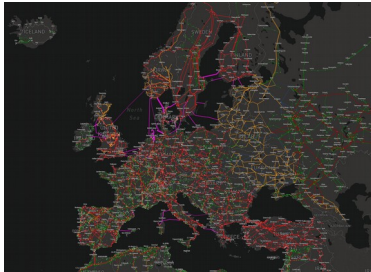
*Efficiency*



*Diffusion speed (rumors, disease, etc.)*

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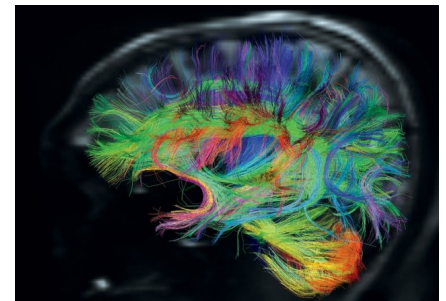
Understand the link between the **structure** of the network and...



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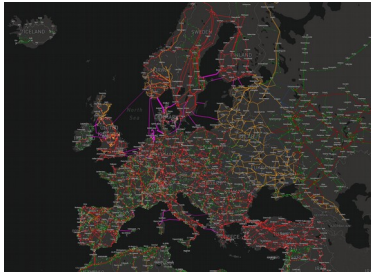
*Diffusion speed (rumors, disease, etc.)*



*Biological function*

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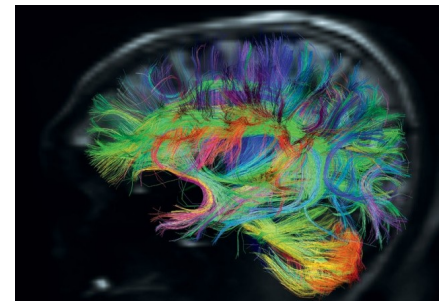
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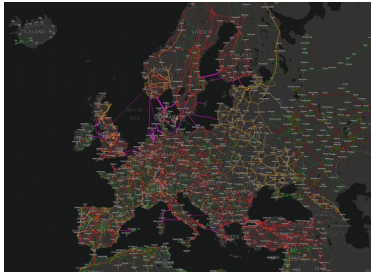
*Traffic bottlenecks*



*Biological function*

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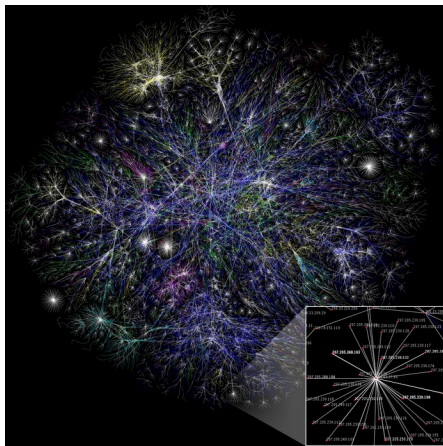
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*Efficiency*



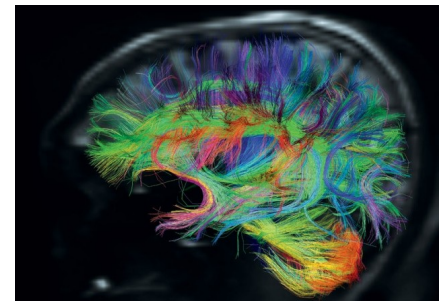
*Diffusion speed (rumors, disease, etc.)*



*Vulnerability*



*Traffic bottlenecks*



*Biological function*

# Why bother studying networks?

## Prototypal questions:

- I have X M€ to invest in Grenoble's public transportation service. Add a new tram station? Add a new bus line?
- I have vaccine shots for only 100 people in a community of  $10^4$  people. Who should I treat first?
- How fast does a rumor spread in a social network? In a pyramidal network as in classical firms? In a more horizontal network as on Twitter?
- A few large powerplants disseminated in the country is not the most efficient structure for an electrical grid. How can we do better?
- I want to monitor the pressure in a water distribution network. Where should I install sensors?
- If I choose a random IP address and attack it. What are the chances that I disconnect Internet?
- Given a transportation network and a list of packages to be delivered in different places, what is the fastest route?
- etc.



# How do we study networks?

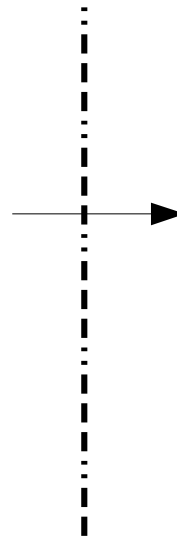
**1/ First step:** modelisation. Model the network as a *graph*.

## Example

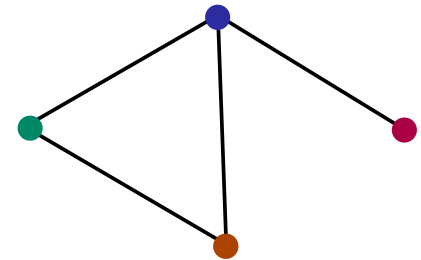
### Real world (network)

Alice, Bob and Charlotte are friends  
Bob is David's father

- Alice, Bob, Charlotte and David are (complex) people
- Their relationships are complex



### Abstract world (graph)



- People are reduced to nodes
- Relationships are reduced to links

# How do we study networks?

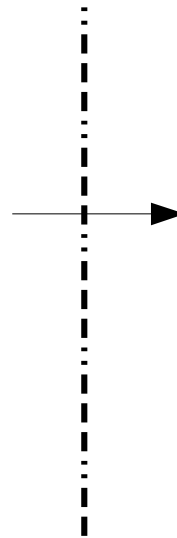
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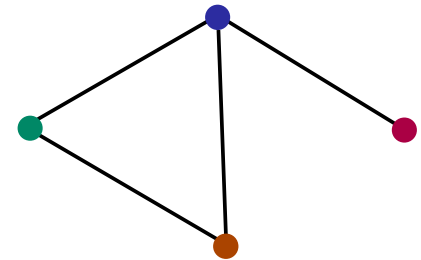
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### Abstract world (graph)



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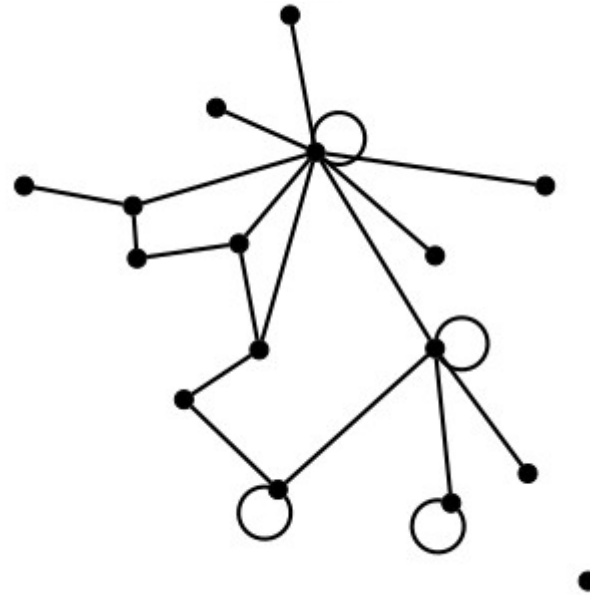
**2/ Second step:** study the obtained graph with mathematical tools from graph theory / network science / computer science / statistics / physics / etc.



# Examples of graphs

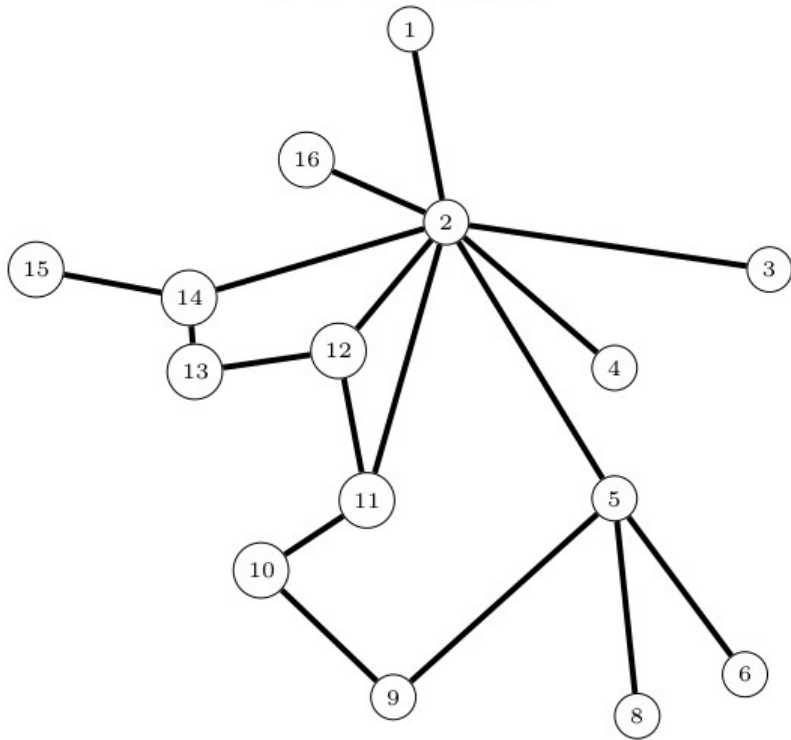
See notebook

# A toy graph



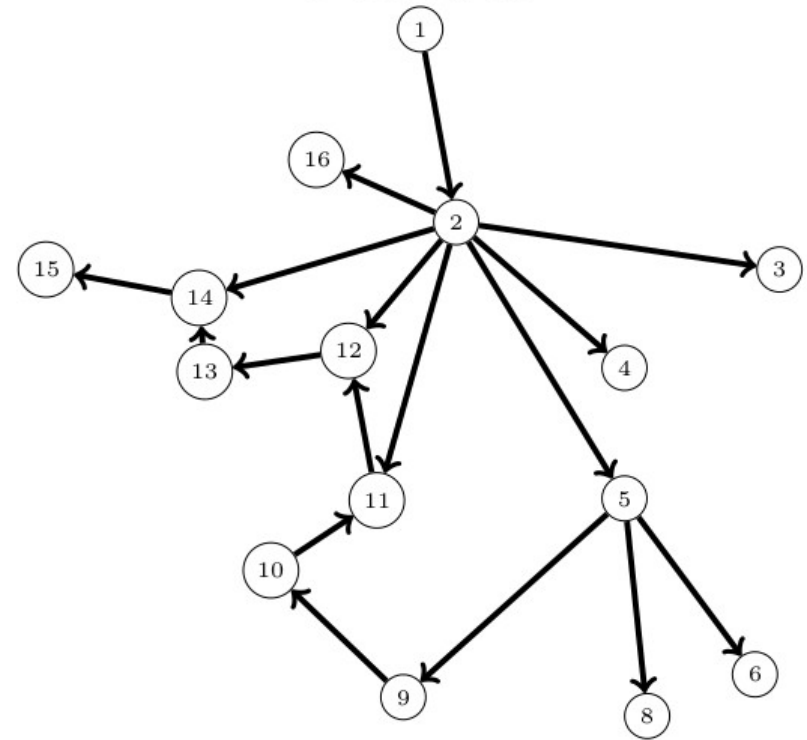
# A toy graph: directed vs undirected

undirected graph



7

directed graph

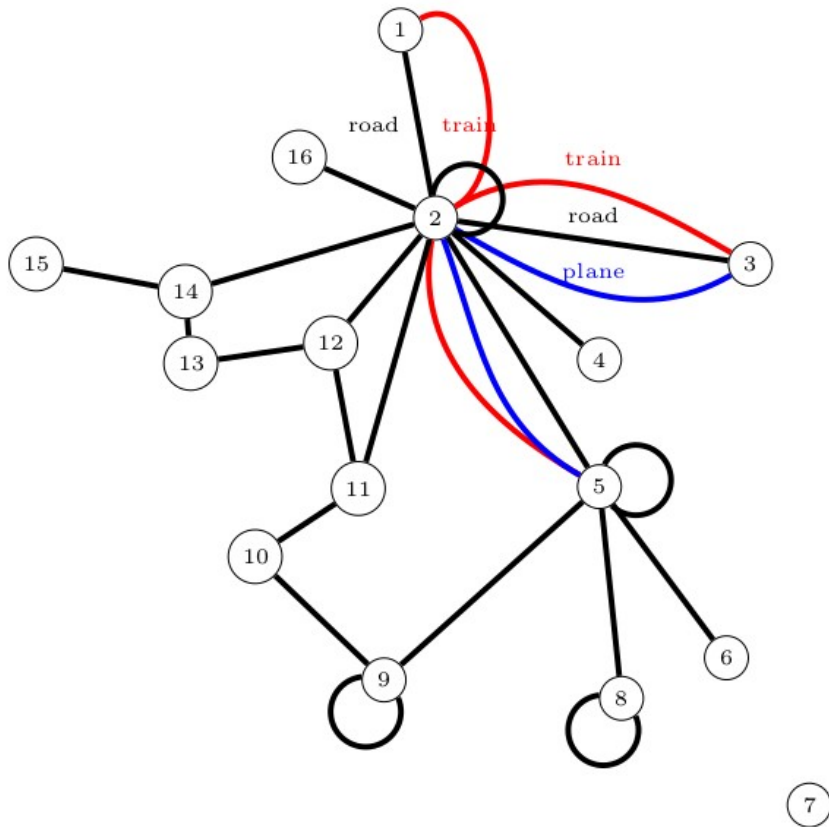


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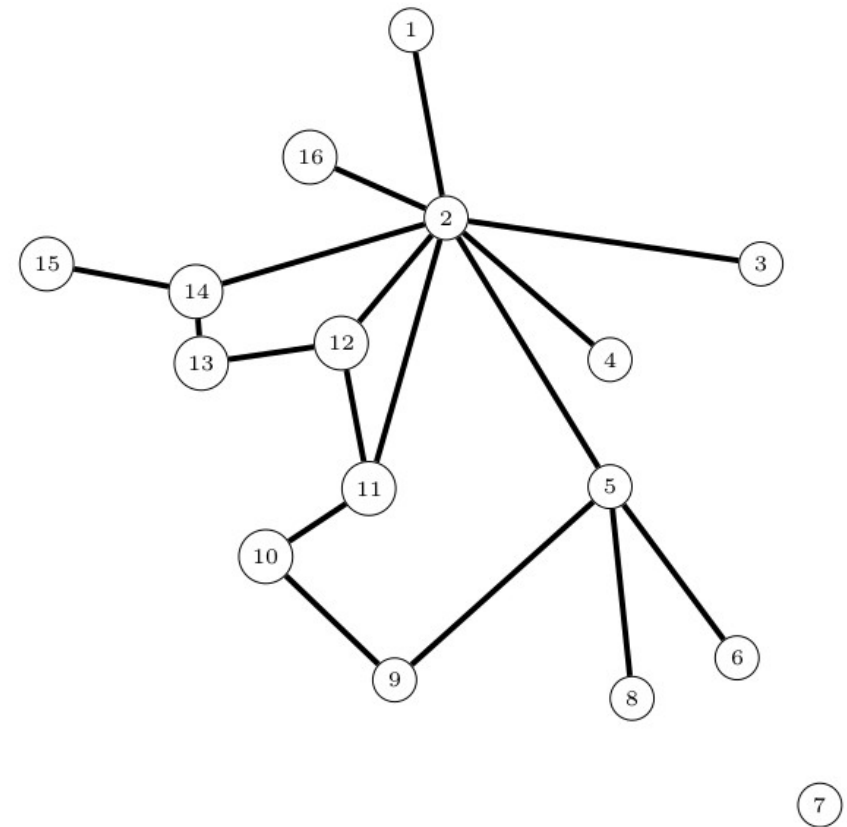


# A toy graph: multi vs simple graph

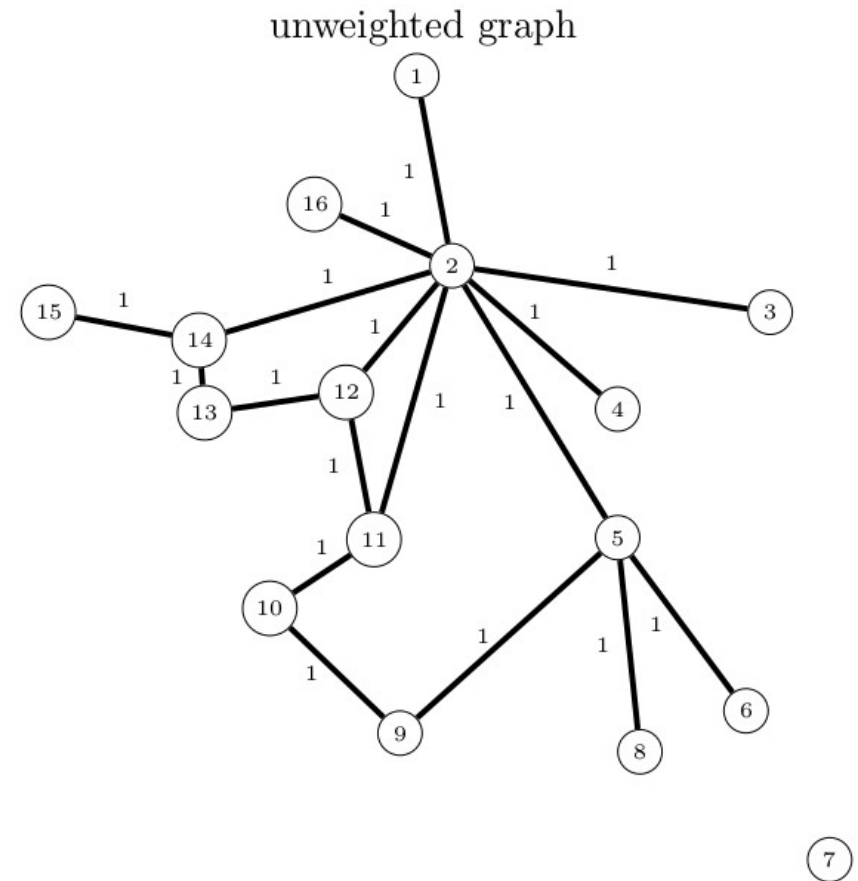
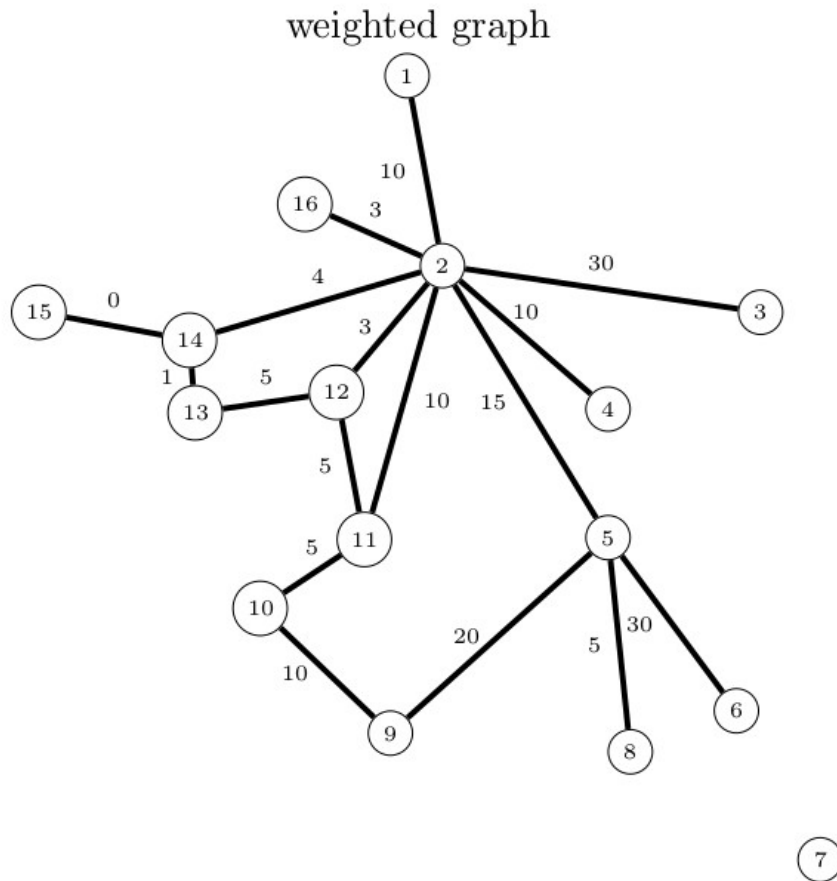
multigraph graph



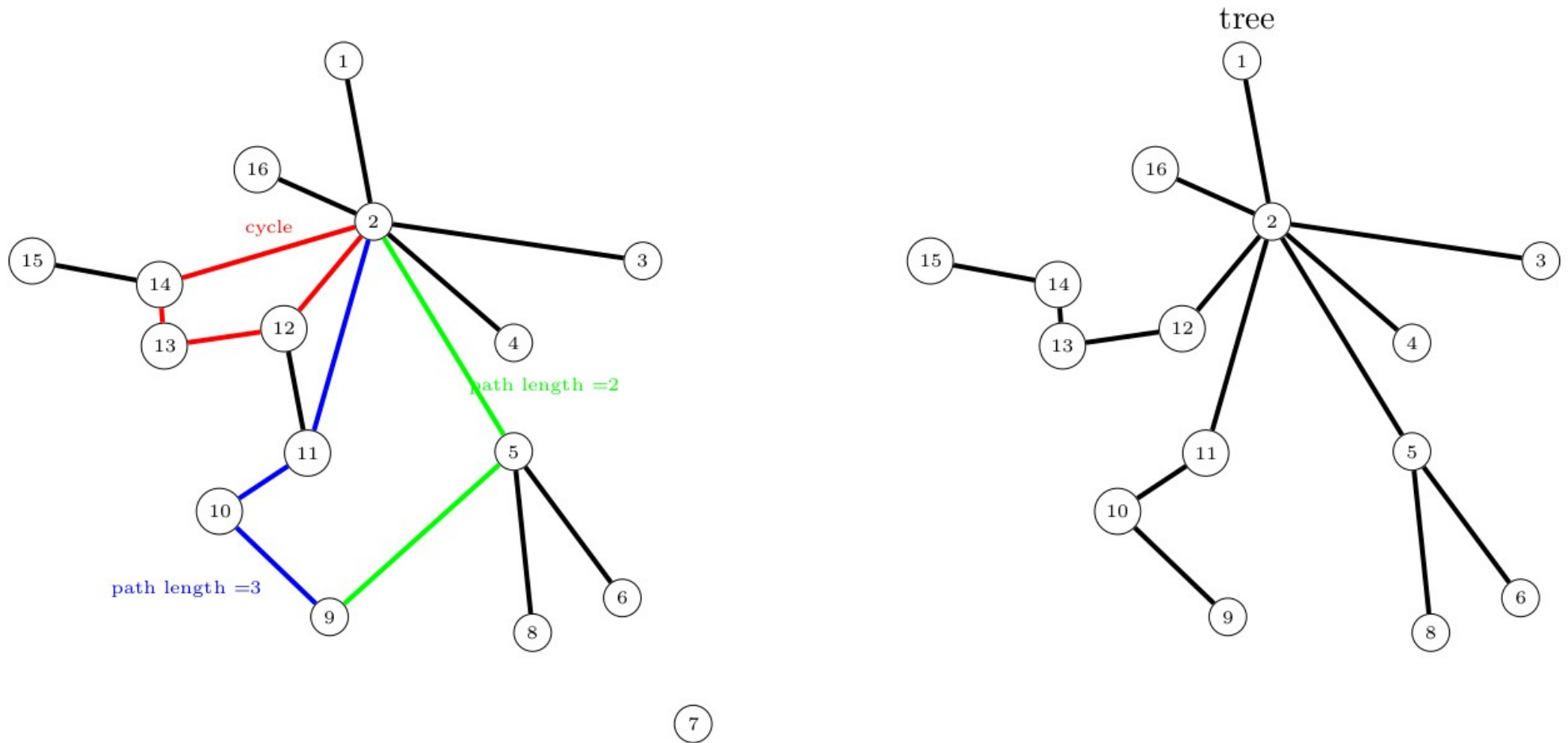
simple graph



# A toy graph: weighted vs unweighted

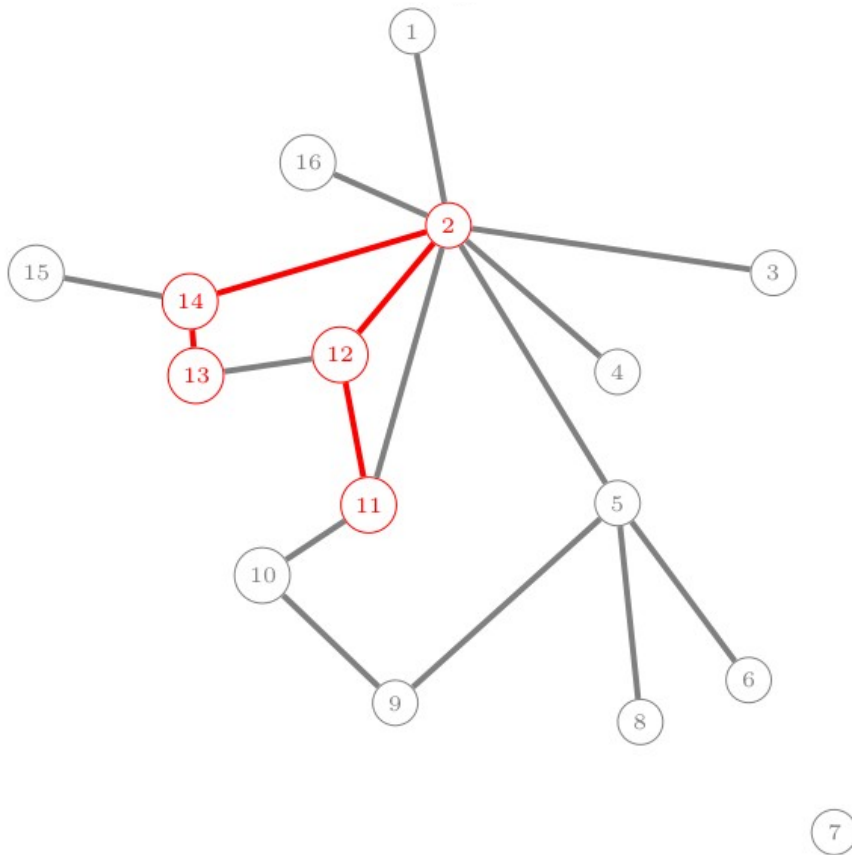


# A toy graph: paths, cycles, trees

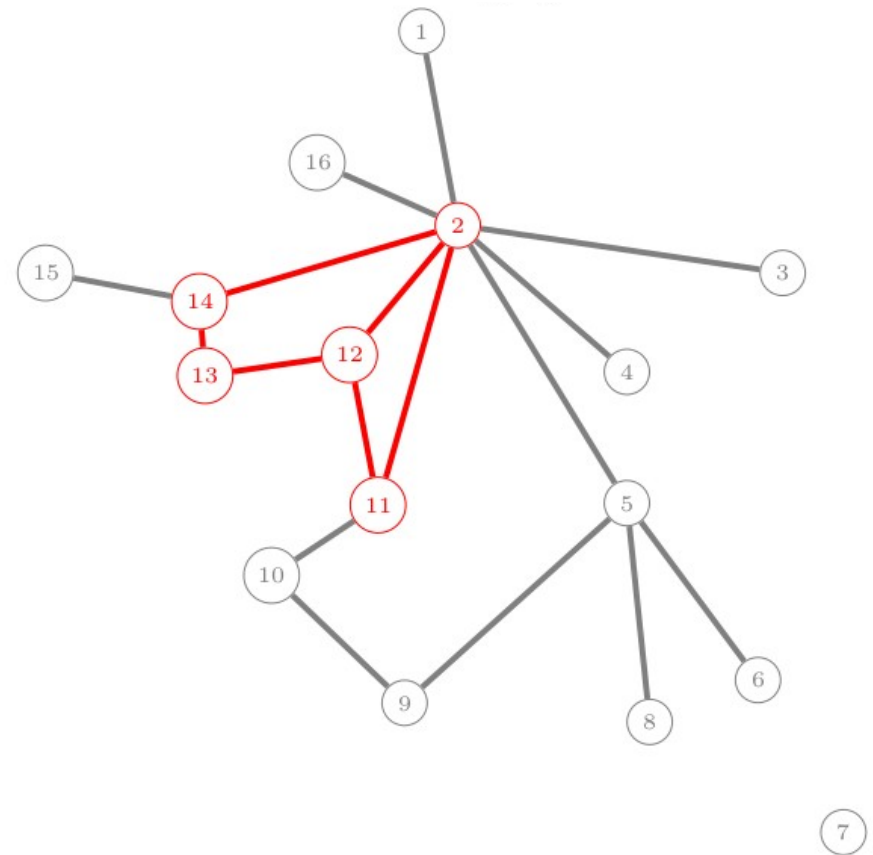


# A toy graph: (induced) subgraph

A subgraph

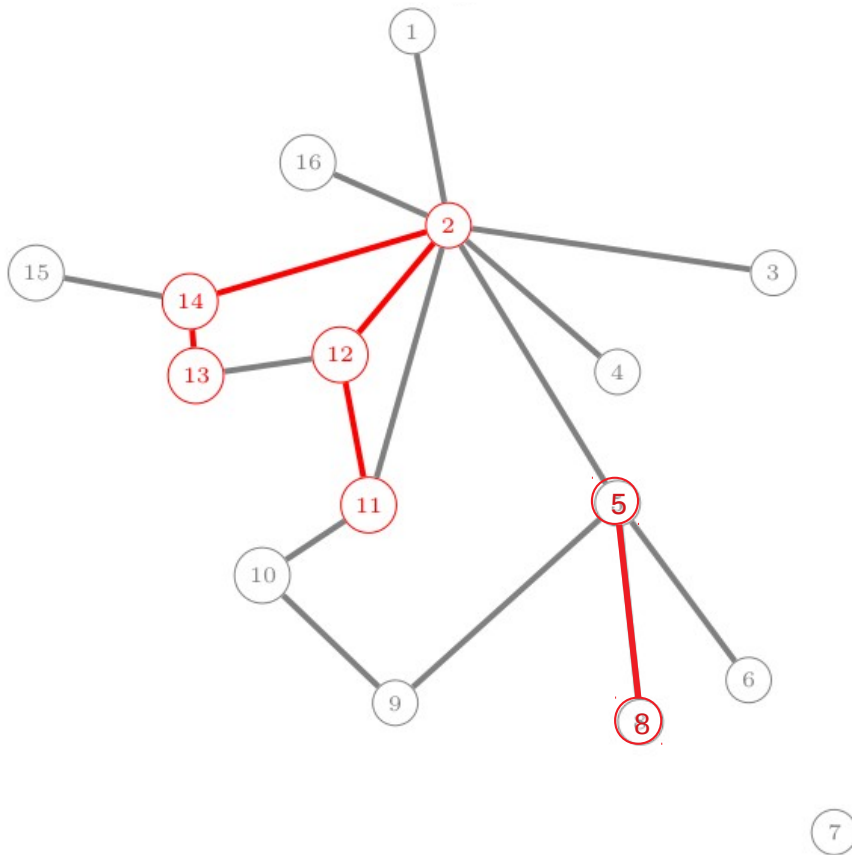


The induced subgraph by nodes {2, 11, 12, 13, 14}

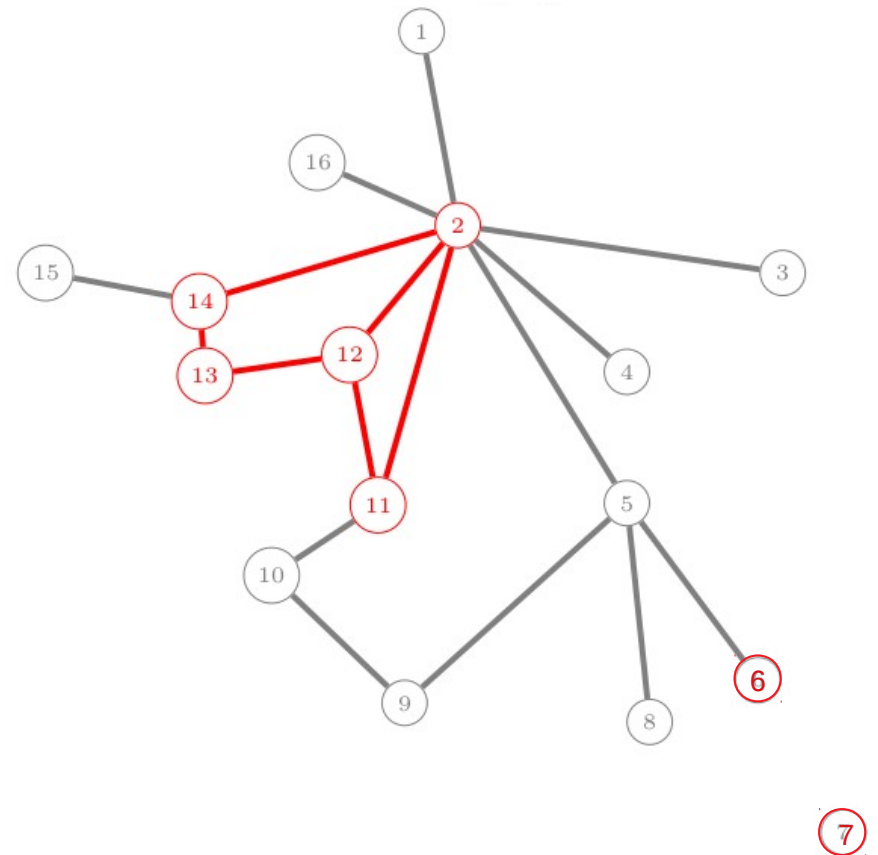


# A toy graph: (induced) subgraph

Another subgraph

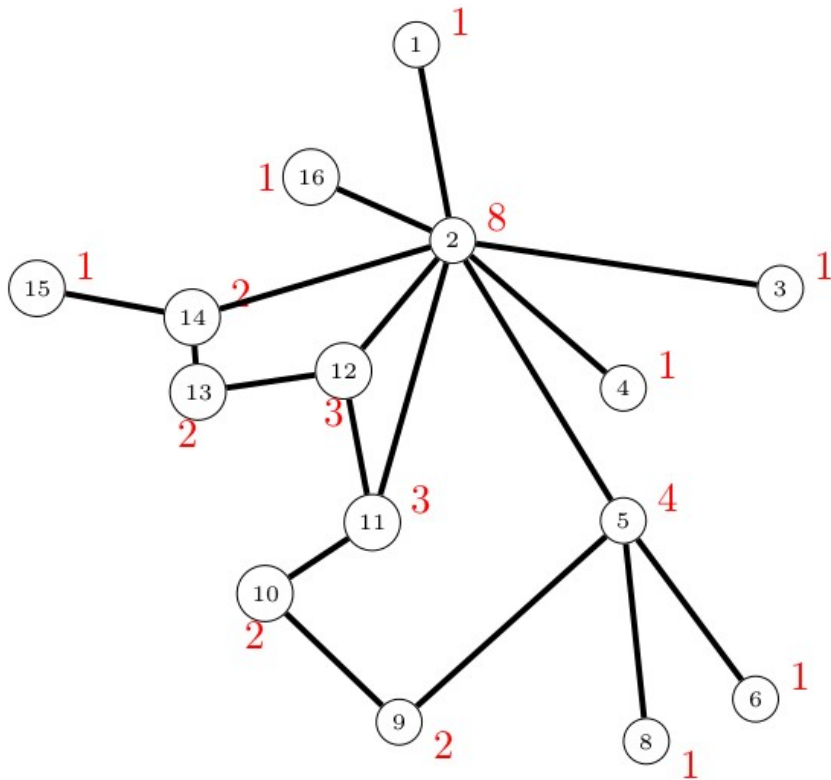


The induced subgraph by nodes {2, 11, 12, 13, 14, 6, 7}

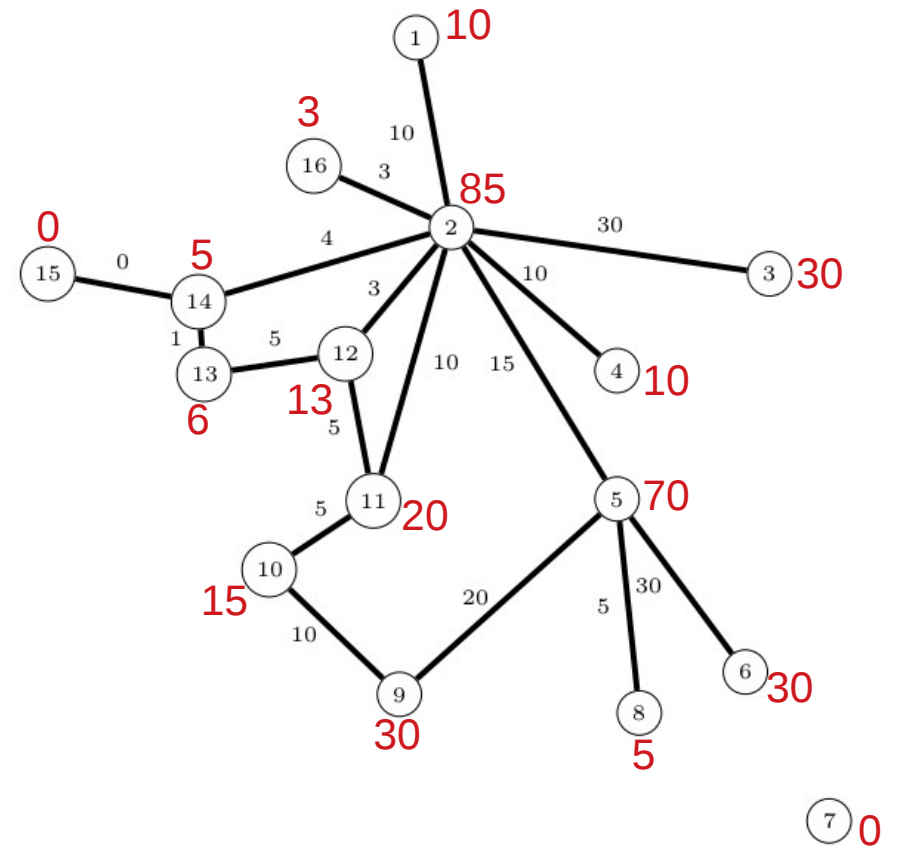




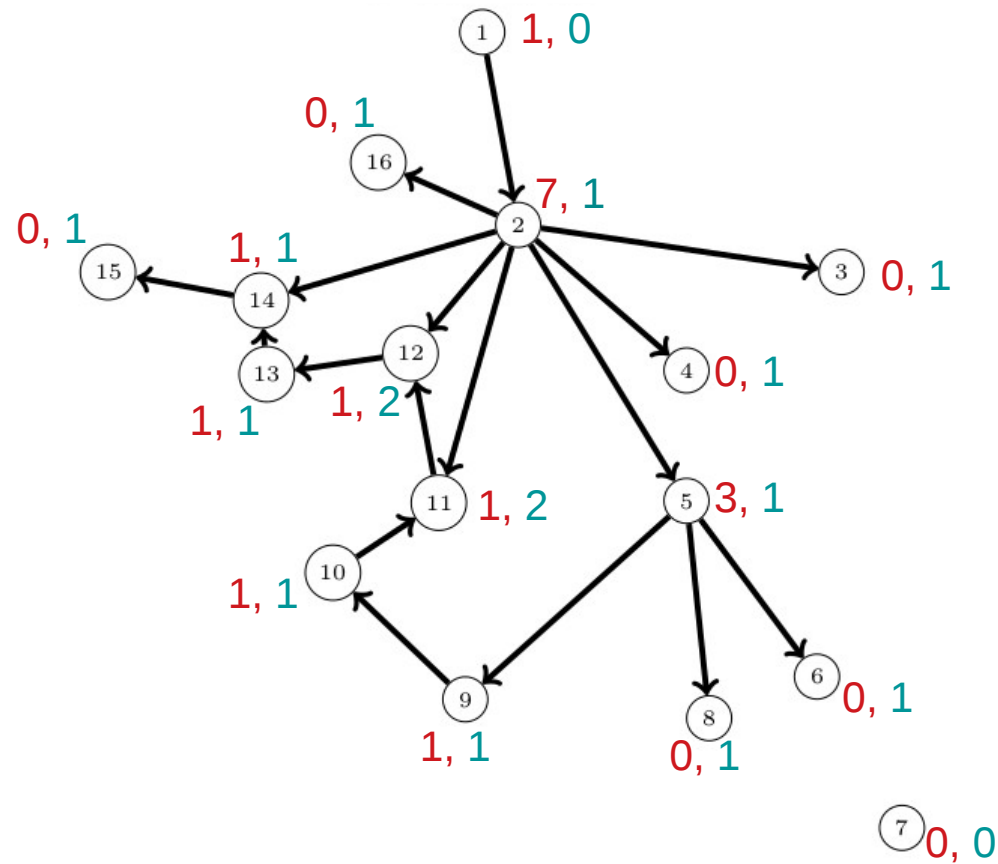
## Degree in unweighted graph



## Degree in weighted graph



# A toy graph: in- and out-degrees in directed graphs



Out-degrees  
In-degrees